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# LAMINAR FILM SURFACE EVAPORATION WITH UNIFORM HEAT FLUX IN A FAST ROTATING DRUM

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**Abstract**—Effective cooling of a fast rotating drum can be obtained by evaporating a liquid flowing as a thin film on a conical inside surface. For special applications, a uniform heat flux and temperature over the outside cylindrical surface of the drum are desired. To obtain this, a wall profile is developed which yields a uniform overall resistance of wall and laminar liquid film.

## NOMENCLATURE

$C$, a constant;
$D$, integration constant;
$F$, dummy variable;
$h$, latent heat of condensation;
$k$, overall heat-transfer coefficient of wall and film;
$L$, total axial flow length of liquid (heated section);
$M$, dimensionless group defined by (11);
$\dot{m}$, local liquid flow rate;
$n$, exponent;
$N$, dimensionless group defined by (7);
$P$, dimensionless group defined by (15);
$p$, pressure;
$\dot{q}$, heat flux per unit area;
$\dot{Q}$, heat flux;
$Re$, Reynolds number of the liquid film;
$r$, inside radius of drum (with index $a$ outside radius);
$T$, temperature;
$u, v$, dummy variables;
$w$, wall thickness;
$x$, axial flow length of liquid film;
$y$, local thickness of liquid film.

Greek symbols

$\beta$, angle between cone and axis;
$\eta$, mean dynamic viscosity of liquid;
$\vartheta$, dimensionless film thickness defined by (9);
$\lambda$, thermal conductivity;
$\nu$, mean kinematic viscosity of liquid;
$\xi$, dimensionless axial flow length defined by (6);
$\rho$, density of liquid;
$\psi$, dimensionless inside drum radius defined by (10);
$\omega$, angular velocity of rotating system.

Subscripts

0, 1, at the points where $\xi = 0$ or $\xi = 1$, respectively;
$a$, on the outside of the cylinder;
$l$, of the liquid;
$n$, normal to the wall;
$s$, at saturation conditions;
$w$, of the wall;
$x$, in the direction of $x$.

## 1. INTRODUCTION

MUCH research and development has been devoted to the estimation of the heat-transfer coefficient with laminar film condensation in rotating or nonuniform gravity systems [1–4]. Recently it was also shown [5, 6] how, in a fast rotating cylinder, a uniform overall resistance of wall and laminar condensate film can be obtained by using a curved, conical condensation surface inside the drum. This type of fast rotating drum may be applied to paper drying, where a high and uniform heat flux from the inside to the outside is desirable.

In other areas of technology, such as metallurgical or plastics technology, fast rotating drums are in contact with a hot material being cooled, and heat flows in the opposite direction. Effective cooling can be obtained by film surface evaporation in the drum, the inverse process of film condensation. Figure 1 shows a construction example where the liquid flows first, through a stationary or rotating tube, into a groove, and then over its edge onto the evaporation surface. The excess fluid which does not evaporate on the way along the inside surface, is collected in a second groove and is sucked off by means of a stationary syphon as proposed in [5].

In this paper it is shown how a uniform overall heat-transfer resistance of wall and laminar liquid film can

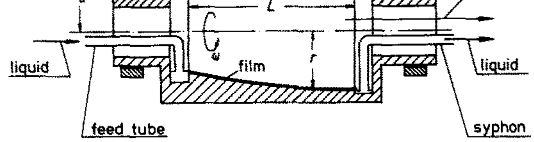


FIG. 1. Cooling drum with curved, conical inside surface.

also be obtained in the case of evaporation by using a specially-curved, conical inside surface.

For convenience, the nomenclature has been taken to be the same as in [5] and [6] wherever possible.

## 2. DEVELOPMENT OF WALL PROFILE

In [5] and [6] it had been shown that, for the calculation of the film thickness, the acceleration forces normal to the wall can be neglected. The normal forces (represented by $b_n$) affect the film thickness only very near the end of the flow path. Only for a cylindrical surface is this simplification not allowed: this limiting case is not being considered here.

Assuming a parabolic Nusselt velocity profile in the film yields, according to (1) of [5] with $b_n = 0$ and the circumference as the width of the flow path,

$$b_x = \frac{3 \cdot \dot{m} \cdot v}{2 \cdot \pi \cdot r \cdot \rho \cdot y^3}. \quad (1)$$

As shown in Fig. 2

$$\frac{b_x}{\omega^2 \cdot r} = \sin\beta \approx \tan\beta = \frac{dr}{dx}. \quad (2)$$

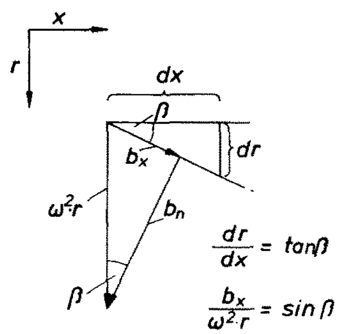


FIG. 2. Inclined inside surface in the acceleration field.

The slope of the cone is assumed to be so small that the simplification $\sin\beta = \tan\beta$ is allowed. Combining (1) and (2) gives

$$\frac{dr}{dx} = \frac{3 \cdot \dot{m} \cdot v}{2 \cdot \pi \cdot \omega^2 \cdot r^2 \cdot \rho \cdot y^3}. \quad (3)$$

Assuming the desired uniform heat flux over the outside surface yields

$$(\dot{m}_0 - \dot{m}) \cdot h = \dot{q}_a \cdot 2 \cdot \pi \cdot r_a \cdot x \quad (4)$$

and at the end of the flow path

$$(\dot{m}_0 - \dot{m}_1) \cdot h = \dot{q}_a \cdot 2 \cdot \pi \cdot r_a \cdot L \quad (5)$$

with the dimensionless variable

$$\xi = \frac{x}{L} \quad (6)$$

and the dimensionless group

$$N = \frac{\dot{m}_0 - \dot{m}_1}{\dot{m}_0} \quad (7)$$

the local flow rate $\dot{m}$ can also be expressed by

$$\frac{\dot{m}}{\dot{m}_0} = 1 - N \cdot \xi \quad (8)$$

with $\dot{m}_1$ in (7) according to (5).

Substituting $\dot{m}$ according to (8) in (3), and introducing the dimensionless variables according to (6) and

$$\vartheta = \frac{y}{y_0} \quad (9)$$

$$\psi = \frac{r}{r_0} \quad (10)$$

as well as the dimensionless group

$$M = \frac{3 \cdot \dot{m}_0 \cdot v \cdot L}{2 \cdot \pi \cdot \omega^2 \cdot \rho \cdot r_0^3 \cdot y_0^3} \quad (11)$$

gives

$$\frac{d\psi}{d\xi} = M \cdot \frac{1 - N \cdot \xi}{\psi^2 \cdot \vartheta^3}. \quad (12)$$

We assume a uniform heat flux $q_a$ and temperature difference $T_{w,a} - T_s$, which can be obtained only by a uniform overall resistance of wall and film. The wall thickness, or the inside radius, respectively, must change in the axial direction in such a way that the overall resistance remains constant. The film thickness is very small compared to the radius, and the mean radius for heat conduction of the film can be assumed to be equal to the inside drum radius $r$. The mean radius of the wall is taken as the logarithmic mean value of the outside and inside radii. The constant overall resistance can be expressed by

$$\frac{y}{r \cdot \lambda_l} + \frac{\ln \frac{r_a}{r}}{\lambda_w} = \text{const.} \quad (13)$$

and the heat flux and temperature difference, for instance at point $\xi = 0$,

$$\dot{q}_a \cdot r_a \cdot \left( \frac{y_0}{r_0 \cdot \lambda_l} + \frac{\ln \frac{r_a}{r_0}}{\lambda_w} \right) = T_{w,a} - T_s \quad (14)$$

Introducing in (13) the dimensionless variables $\vartheta$ and $\psi$, (9) and (10), and the dimensionless group

$$P = \frac{r_0}{y_0} \cdot \frac{\lambda_l}{\lambda_w} \quad (15)$$

gives

$$\frac{\vartheta}{\psi} - P \cdot \ln \psi = C. \quad (16)$$

At the point $\xi = 0, \vartheta = \vartheta_0 = 1$ and $\psi = \psi_0 = 1$, which gives $C = 1$, and (16) becomes

$$\vartheta = \psi \cdot (P \cdot \ln \psi + 1). \quad (17)$$

Substituting in (12) $\vartheta$ according to (17) and separating the variables gives

$$F = \int \psi^5 \cdot (P \cdot \ln \psi + 1)^3 \cdot d\psi + D$$
$$= \int M \cdot (1 - N \cdot \xi) \cdot d\xi \quad (18)$$

with the integration constant $D$ for both integrals.

Integrating the second integral in (18) gives

$$F = M \cdot \left( \xi - \frac{N}{2} \cdot \xi^2 \right). \quad (19)$$

Integrating the first integral in (18) gives finally, as shown in the Appendix,

$$F = \frac{\psi^6}{6} \cdot \left[ (P \cdot \ln \psi + 1)^3 - \frac{P}{2} (P \cdot \ln \psi + 1)^2 + \frac{P^2}{6} \cdot (P \cdot \ln \psi + 1) - \frac{P^3}{36} \right] + D. \quad (20)$$

Taking into account that for $\xi = 0$, $\psi = \psi_0 = 1$, and comparing (20) with (19), gives

$$D = -\tfrac{1}{6} \left( 1 - \frac{P}{2} + \frac{P^2}{6} - \frac{P^3}{36} \right). \quad (21)$$

Replacing $F$ in (20, 21) according to (19), yields an equation for $\psi$ and $\xi$.

In the limiting case where $P = 0$ (very high thermal conductivity of the wall material), this equation can be solved for $\psi$, yielding

$$\psi = \left[ 6 \cdot M \cdot \xi \cdot \left( 1 - \frac{N}{2} \cdot \xi \right) + 1 \right]^{1/6}. \quad (22)$$

In the general case where $P > 0$, the combined equations (19) and (20), (21) can only be solved for $\xi$, resulting in

$$\xi = \frac{1}{N} - \sqrt{\left( \frac{1}{N^2} - \frac{2 \cdot F}{M \cdot N} \right)} \quad (23)$$

where $F$ has to be calculated from (20, 21).

To illustrate the application of the equations developed above a numerical example is presented in the following.

## 3. NUMERICAL EXAMPLE

A uniform heat flux of

$$\dot{q}_a = 40\,\text{kW/m}^2 \quad (24)$$

shall be transferred at the cylindrical outside surface of a drum with the dimensions

$$\left. \begin{aligned} L &= 1.00\,\text{m} \\ r_a &= 0.30\,\text{m} \\ r_0 &= 0.25\,\text{m} \end{aligned} \right\} \quad (25)$$

rotating with an angular velocity

$$\omega = 30 \cdot 1/\text{s}. \quad (26)$$

The total heat flux is then according to (24) and (25)

$$\dot{Q} = 2 \cdot \pi \cdot 0.3 \cdot 40 \cdot \text{kW} = 75.40 \quad (27)$$

The wall material is copper with a thermal conductivity [7] of

$$\lambda_w = 372\,\text{W/m°C}. \quad (28)$$

On the inside surface water [8] is evaporated at the saturation conditions

$$\left. \begin{aligned} T_s &= 150°\text{C} \\ p_s &= 4.76\,\text{bar}. \end{aligned} \right\} \quad (29)$$

The relevant properties are [8]

$$\left. \begin{aligned} h &= 2112.6\,\text{kJ/kg} \\ \eta &= 18.6 \cdot 10^{-5}\,\text{kg/ms} \\ 1/\rho &= 0.001091\,\text{m}^3/\text{kg} \\ \lambda_l &= 0.684\,\text{W/m°C} \end{aligned} \right\} \quad (30)$$

and the kinematic viscosity

$$\nu = \eta/\rho. \quad (31)$$

The film thickness at the start of the flow path is taken as

$$y_0 = 0.1\,\text{mm} \quad (32)$$

which gives, according to (14) and the data given above, the driving temperature difference

$$T_{w,a} - T_s = 12.90°\text{C} \quad (33)$$

and the uniform overall heat-transfer coefficient related to the outside surface

$$k_a = \frac{\dot{q}_a}{T_{w,a} - T_s} = 3101 \frac{\text{W}}{\text{m}^2\,°\text{C}}. \quad (34)$$

With the data from (25), (28), (30) and (32) the dimensionless group $P$ can be calculated according to (15) yielding

$$P = 4.597. \quad (35)$$

From (5) and the data according to (24), (25) and (30), one finds the evaporating liquid rate

$$\dot{m}_0 - \dot{m}_1 = 0.03569\,\text{kg/s} \quad (36)$$

which is equal to the ratio $\dot{Q}/h$. The outlet flow rate $\dot{m}_1 \geqslant 0$, and the inlet flow rate is taken as

$$\dot{m}_0 = 0.04000\,\text{kg/s}. \quad (37)$$

Dividing (36) by (37) gives $N$ according to (7)

$$N = 0.8922 \quad (38)$$

Introducing in (11) the values given in (25), (26), (30) to (32) and (37), gives

$$M = 0.3007. \quad (39)$$

With the numerical values of $M, N$ and $P$ according to (39), (38) and (35), the equations (20), (21) and (23) can be applied. For various values of $\psi$, $F$ is first calculated using (20) and (21), and with this value the dimensionless flow length $\xi$ is calculated using (23). From the dimensionless values of $\xi$ and $\psi$ the actual axial flow length $x$ and radius $r$ are calculated according to (6) and (10). The wall thickness

$$w = r_a - r. \quad (40)$$

The results of the calculation are given in Table 1 and Fig. 3. The high overall heat-transfer coefficient according to (34) is obtained with a relatively flat wall

Table 1. Calculated values of wall profile

| $\xi$ | $\psi$ | $x$ (mm) | $r$ (mm) | $w$ (mm) |
|---|---|---|---|---|
| 0 | 1.000 | 0 | 250.00 | 50 |
| 0.03685 | 1.010 | 36.85 | 252.50 | 47.50 |
| 0.08340 | 1.020 | 83.40 | 255.00 | 45.00 |
| 0.14115 | 1.030 | 141.15 | 257.50 | 42.50 |
| 0.21413 | 1.040 | 214.13 | 260.00 | 40.00 |
| 0.30807 | 1.050 | 308.07 | 262.50 | 37.50 |
| 0.43395 | 1.060 | 433.95 | 265.00 | 35.00 |
| 0.61823 | 1.070 | 618.23 | 267.50 | 32.50 |
| 0.64227 | 1.071 | 642.27 | 267.75 | 32.25 |
| 0.75878 | 1.075 | 758.78 | 268.75 | 31.25 |
| 0.89093 | 1.078 | 890.93 | 269.50 | 30.50 |
| 0.95886 | 1.079 | 958.86 | 269.75 | 30.25 |

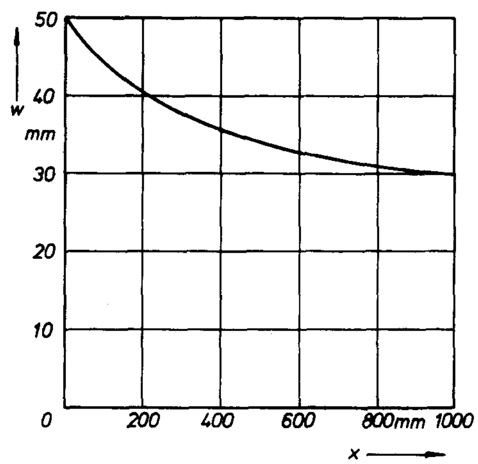


FIG. 3. Calculated wall profile for uniform overall resistance of wall and film.

profile. The total change of wall thickness is about 20 mm. The maximum slope occurs at the start of the flow path. Examining (12) and the definitions (6) and (10) of $\xi$ and $\psi$, reveals that at the point $\xi = 0$

$$\left(\frac{dr}{dx}\right)_0 = M \cdot \frac{r_0}{L} = 0.07517 \tag{41}$$

and the maximum relative error of $b_x$, caused by the assumption made in (2), is +0.28%. This means that the assumption ($\sin\alpha = \tan\alpha$) is permissible.

Further, it was assumed that the film is in laminar flow. The maximum Reynolds number occurs at the start of the flow path. With the data from (25), (30) and (37) the Reynolds number

$$Re_0 = \frac{\dot{m}_0}{2 \cdot \pi \cdot r_0 \cdot \eta} = 136.9. \tag{42}$$

Thus the assumption of laminar flow is also justified (if the effect of gravity is negligible).

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## APPENDIX

The integral can be solved by successive partial integration according to

$$\int u \cdot dv = u \cdot v - \int v \, du \tag{A1}$$

with

$$u = (P \cdot \ln\psi + 1)^n \tag{A2}$$

and

$$dv = \psi^5 \cdot d\psi. \tag{A3}$$

Integrating (A3) yields

$$v = \tfrac{1}{6} \cdot \psi^6 \tag{A4}$$

and differentiating (A2) gives

$$du = \frac{n \cdot P}{\psi} \cdot (P \cdot \ln\psi + 1)^{n-1} \cdot d\psi. \tag{A5}$$

Introducing (A2)–(A5) into (A1) gives

$$\int (P \cdot \ln\psi + 1)^n \cdot \psi^5 \cdot d\psi = (P \cdot \ln\psi + 1)^n \cdot \tfrac{1}{6} \cdot \psi^6 - \frac{P \cdot n}{6} \cdot \int (P \cdot \ln\psi + 1)^{n-1} \cdot \psi^5 \cdot d\psi. \tag{A6}$$

Starting with $n = 3$ and applying (A6) three times, yields finally equation (20) of this paper.

# EVAPORATION A LA SURFACE D'UN FILM LAMINAIRE A FLUX THERMIQUE CONSTANT DANS UN TAMBOUR EN ROTATION RAPIDE


**Résumé**—Le refroidissement efficace d'un tambour en rotation rapide peut être obtenu par évaporation d'un liquide s'écoulant en film mince sur une surface conique intérieure. Pour certaines applications on désire un flux thermique et une température uniformes sur la surface cylindrique extérieure du tambour. Pour l'obtenir, on détermine un profil de paroi qui présente une résistance globale uniforme comprenant la paroi et le film liquide laminaire.

## LAMINARE FILMOBERFLÄCHENVERDAMPFUNG MIT EINHEITLICHEM WÄRMEFLUSS IN EINER SCHNELL ROTIERENDEN TROMMEL

**Zusammenfassung**—Schnell rotierende Trommeln können wirksam gekühlt werden, indem man innen eine Flüssigkeit verdampft, die als dünner Film über eine konische Innenfläche strömt. Für besondere Anwendungen sind an der zylindrischen Außenfläche der Trommel ein einheitlicher Wärmefluß und eine einheitliche Temperatur erwünscht. Um dieses zu erreichen, wird ein Wandprofil entwickelt, welches einen einheitlichen Gesamtwiderstand von Wand und laminarem Flüssigkeitsfilm liefert.

## ЛАМИНАРНОЕ ИСПАРЕНИЕ ПЛЕНКИ С ПОВЕРХНОСТИ БЫСТРО ВРАЩАЮЩЕГОСЯ БАРАБАНА ПРИ УСЛОВИЯХ ОДНОРОДНОГО ТЕПЛОВОГО ПОТОКА СТЕНКИ

**Аннотация** — Рассматривается задача эффективного охлаждения быстро вращающегося барабана испарения жидкости из тонкой пленки, стекающей по конической внутренней поверхности. Для специальных приложений требуются условия однородности теплового потока и температуры на внешней цилиндрической поверхности. Для достижения этой цели выбирается профиль стенки, обеспечивающий общее однородное термическое сопротивление стенки и ламинарной жидкой пленки.